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The effect of radiation on the laminar natural convection induced by a line heat source

Mohammad B. Ayani and Javad A. Esfahani
Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran, and
Antonio C.M. Sousa
Department of Mechanical Engineering, University of New Brunswick, Fredericton, Canada

Abstract

Purpose – To show the effect of radiation from the heat source and the variation of fluid properties on the laminar natural convection induced by a line heat source.

Design/methodology/approach – The governing equations – Navier-Stokes and energy equation are discretized in a staggered grid by a control volume approach, and they are solved using a segregated technique. The equations for the fluid and solid (line heat source) phases are solved simultaneously. The three sides of the computational domain are open boundary. Some of the physical and thermo-physical properties of the fluid (air) such as density, thermal conductivity and viscosity were considered to vary with temperature.

Findings – The present predictions are compared with those using the Boussinesq approximation, with the results for the boundary layer equations, and with the experimental results. The present predictions reveal considerable departure from the Boussinesq-based solution and from the boundary layer results. This study also shows the radiation exchange between the heat source and surrounding has major effect in the results. Thus, the departure between the experimental and analytical results can be explained by the effect of radiation exchange.

Research limitations/implications – In this work, just studied steady-state laminar thermal plume with the effects of radiation from heat source and the variation of air properties with temperature while it is propose to extend this work to transient and/or turbulent flow.

Originality/value – The effect of radiation from a line heat source on the flow filed around the source and offers enhancement of design to thermal engineers.

Keywords Numerical analysis, Convection, Heat transfer, Fluids

Paper type Research paper

Nomenclature

c_p = specific heat
f = dimensionless stream function
g = gravitational acceleration
Gr = Grashof number
h = dimensionless temperature
H = height of computational domain
k = thermal conductivity
p = pressure
Pr = Prandtl number
$\dot{q}_{gen}$ = heat generation

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q_{rad} = radiation exchange
Q = rate of heat transfer
T = temperature
u = velocity component in the x direction
v = velocity component in the y direction
$\vec{V}$ = velocity vector
x = coordinate along vertical direction
X = body force in the x direction
y = coordinate along horizontal direction
W = width of computational domain

Greek symbols
β = thermal expansion coefficient
ε = emissivity
μ = dynamic viscosity
ν = kinematics viscosity
ρ = density
σ = normal stress
σ = Stefan-Boltzmann constant
τ = shear stress
ξ = similarity variable
ψ = stream function

Subscripts
0 = centerline
f = fluid phase
s = solid phase
con = convection
tot = total
∞ = ambient

1. Introduction

Natural convection induced by heat sources in an infinite fluid space is relevant in many engineering applications, and, in particular, laminar natural convection generated by horizontal line heat sources, and also from heated circular cylinders, and it has been extensively investigated analytically, numerically and experimentally. Notwithstanding, a few questions still remain unanswered, and this work addresses the validity of the boundary layer-type equations, and the factors affecting the accuracy of the numerical solutions, including the Boussinesq approximation, variable properties, and radiation exchange.

Zeldovich (1937) in his pioneering work described the natural convection plumes arising from a point and from a horizontal line source of heat. The treatment used does not allow a velocity component normal to the symmetry plane of plume. Fujii (1963) solved the two-dimensional boundary layer equations for Pr of 0.01, 0.7, 2 and 10 by using a similarity approach, which reduces the set of four partial differential equations into two ordinary differential equations. It should be mentioned that many experimental papers use Fujii's paper for comparison and validity assessment. Gebhart *et al.* (1970) redefined some of the variables used by Fujii, and simplified the flow governing equation in the boundary layer region. For instance, the Grashof number was defined based on the difference between the centerline temperature of the plume and the ambient temperature – Fujii had defined it based on the total heat transfer to the plume. Jaluria and Gebhart (1977) studied laminar natural convection flow arising from a steady line thermal source, which is positioned at the leading edge of a vertical adiabatic surface. The two-dimensional boundary layer flow equations were reduced to self-similarity equations, and they were solved numerically. Lin *et al.* (1996) examined the inclined wall plumes that arise from a line thermal source embedded at the leading edge of an adiabatic plate with arbitrary tilt angle. They carried out both experiments, and numerical analyses based on the self-similarity equations.

The laminar plume rising above a long electrically heated wire of small diameter was studied experimentally by Brodowicz and Kierkus (1966). They measured the velocity and temperature distribution in air above the wire with a length to diameter ratio (L/D) of 3,330. Their results only show a fair agreement with the results produced using the boundary layer theory (Fujii, 1963; Gebhart *et al.* 1970). Forstrom and Sparrow (1967) use a thermocouple to measure the temperature distribution in air at various heat inputs and heights above a wire source. Also, Schorr and Gebhart (1970) measured the temperature distribution in a plume above a wire, which is submerged in light silicon oil. All experimental results consistently present a lower value for the centerline temperature of the plume than that predicted by the similarity solutions, as discussed by Gebhart *et al.* (1988). Linan and Kurdymov (1998) studied numerically the laminar free convection induced by a line heat source at small Grashof numbers, using the Boussinesq equations, in stream function-vorticity (ψ-ω) variables.

A few numerical studies were conducted to analyse the plume arising above heated horizontal circular cylinders. Kuehn and Goldstein (1980) studied the laminar natural convection heat transfer from a horizontal isothermal cylinder by solving the Navier-Stokes and energy equations in (ψ-ω) form. Shin and Chang (1989) studied the transient natural convection from a horizontal circular cylinder subjected to a sudden temperature change. They solved numerically the (ψ-ω) form of the Navier-Stokes and energy equations. An interesting finding of this study was an overshoot in the heat transfer coefficient, which seemed to be associated with the conduction-convection transition, and its magnitude was influenced by the *Pr* and *Ra* numbers. Wang *et al.* (1991) studied the transient laminar natural convection from horizontal cylinders, and they used the ψ-ω equations. Esfahani and Sousa (1999) in studies addressed to ignition by radiation used a primitive variable segregated numerical method based on Patankar (1980) to analyse the laminar thermal plume up to ignition threshold. Their predictions for the ignition delay are in good agreement with published experimental data.

In the above-mentioned analytical and numerical studies except Esfahani and Sousa (1999), the radiation exchange between the heat source and surrounding is ignored; constant values for the viscosity and thermal conductivity were used, along with the Boussinesq approximation for the estimation of the body force in the momentum equations. Also in these studies, with exception of Esfahani and Sousa (1999), the equations for self-similarity, or the ψ-ω and energy equation were solved, while in this study, the primitive variable, steady state Navier-Stokes and energy equations with the effect of radiation exchange between heat source and surrounding are solved for the fluid and solid (line heat source) phases, simultaneously by a segregated numerical method Patankar (1980). The convective-diffusive linkage in this study, uses the power-law scheme Patankar (1980). The governing equations were discretized for a non-uniform staggered grid and the physical and thermo-physical properties of fluid such as density, thermal conductivity and viscosity were considered to be dependent upon the temperature.

2. Mathematical formulation

2.1 Governing equations

Laminar natural convection flow from a horizontal line heat source, assuming the end-effects of the source are negligible, is governed by the continuity equation, the two-dimensional Navier-Stokes equations and the energy equation. Based on the physical configuration shown in Figure 1, the governing equations in the Cartesian coordinate system take the following form:

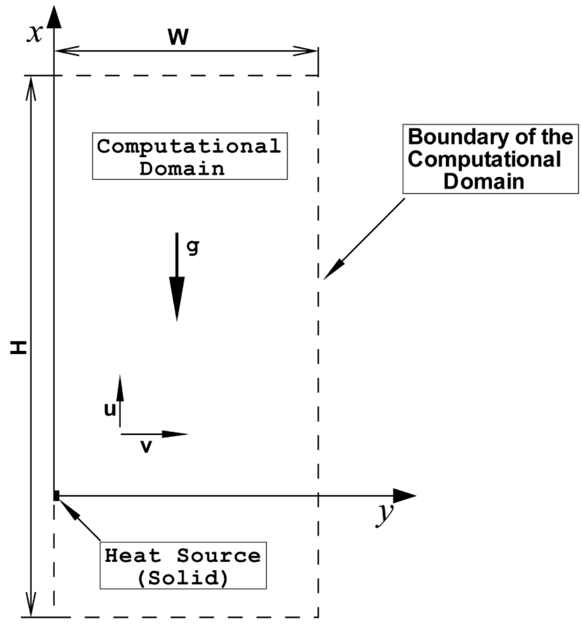


Figure 1.
Schematic of the computational domain and the Cartesian coordinate system

Continuity equation

$$\nabla\cdot(\rho\vec{V}) = 0 \tag{1}$$

where

$$\vec{V} = u\hat{i} + v\hat{j} \tag{2}$$

Momentum in x direction

$$\frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = X - \frac{\partial p}{\partial x} + \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}\right) \tag{3}$$

Momentum in y direction

$$\frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} = -\frac{\partial p}{\partial y} + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y}\right) \tag{4}$$

where

$$\sigma_x = -\frac{2}{3}\mu\nabla\cdot\vec{V} + 2\mu\frac{\partial u}{\partial x} \tag{5}$$

$$\sigma_y = -\frac{2}{3}\mu\nabla\cdot\vec{V} + 2\mu\frac{\partial v}{\partial y} \tag{6}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{7}$$

X in equation (3) is the body force per unit volume in the x direction.

Energy equation

$$\frac{\partial(\rho u T)}{\partial x} + \frac{\partial(\rho v T)}{\partial y} = \frac{1}{c_p}\nabla\cdot(k\nabla T) + \frac{\dot{q}_{gen}}{c_p} \tag{8}$$

where $\dot{q}_{gen}$ is the rate of heat generation per volume in the solid phase.

The flow is assumed to be incompressible in what concerns the variation of density with pressure, therefore, the variation of density of the fluid (air) with temperature can be determined from the following relation:

$$\rho T = \rho_\infty T_\infty \tag{9}$$

and the variation of viscosity of air with temperature is determined from Sutherland's law (Anderson, 1991), as follow:

$$\mu = 1.458 \times 10^{-6} \frac{T^{3/2}}{T + 110.4\,\mathrm{K}} \left(\frac{Ns}{m^2}\right) \tag{10}$$

The variation of c_p and Pr of air with temperature is nearly negligible and they can be considered to have a constant value in the range of temperatures for which the computations are carried out. Under this assumption the variation of the thermal conductivity for air with temperature can be determined from:

$$Pr = \frac{c_p \mu}{k} \tag{11}$$

This formulation for Pr establishes a direct relation between k and μ. The values of μ and k in the temperature range of the present work, i.e. 273-373 K, were calculated from equations 10 and 11. Comparison of these values with the tabulated properties of air (Incropera and DeWitt, 2002) for this particular temperature range indicates a maximum error around 1 percent.

2.2 Boundary conditions

The flow is symmetric about a vertical plane passing through the axis of the heat source (Figure 1), therefore, only one half plane will be considered. The boundary conditions for the symmetry plane ($y = 0$) are as follows:

$$v = 0 \text{ and } \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0 \tag{12}$$

The other boundaries are located relatively far away from the heat source, and the pressure is assumed to have a constant value. In this study, the relative pressure is taken as zero. The tangential velocity component at the boundaries is determined by solving the respective momentum equation in the domain, while the normal velocity component towards the boundary can be obtained by using the extrapolation proposed in Versteeg and Malalasekera (1995).

Two types of conditions for the energy equation are used at the "far-field" boundaries, namely:

$$T = T_\infty \quad \text{(inflow)} \tag{13}$$

$$\frac{\partial T}{\partial n} = 0 \quad \text{(outflow)} \tag{14}$$

where n is the direction perpendicular to the surface of the boundaries.

The boundary condition for the energy equation at the interface of the solid and fluid phases is as follows:

$$-k_{\mathrm{s}} \frac{\partial T_{\mathrm{s}}}{\partial n} = -k_{\mathrm{f}} \frac{\partial T_{\mathrm{f}}}{\partial n} + \varepsilon\sigma\left(T_s^4 - T_\infty^4\right) \tag{15}$$

In this equation, it is assumed the fluid (air) is a non-participating media and n is the direction perpendicular to the interface surface of the solid and fluid phases.

3. Numerical method

The solid heat source is represented in the two-dimensional domain by a square with a side of 0.001 m, and its thermal conductivity is chosen equal to 0.2 W/m K. The center of the square is placed coincident with the origin of the coordinate system as shown in Figure 1.

The numerical method used to solve the momentum equations accounts for the pressure-velocity coupling by iterating the solution, according to an algorithm based on the simple approach Patankar (1980). In the discretization of non-linear convection terms, as already stated, the power-law scheme is used, which, based on extensive numerical tests, was found not to be particularly computationally demanding, and to provide an extremely good representation of the exponential behavior of the convection-diffusion equations.

The governing equations were discretized for non-uniform staggered grids, which reach their smallest value at the fluid-solid interface. The discretized equations for fluid and solid (line source) phases were solved simultaneously. The values of the components velocity in the solid phase are set zero by choose of suitable source terms in the discretized momentum equations (Patankar, 1980). Equation (15) was formulated in the following form:

$$-k_{\mathrm{s}} \frac{\partial T_{\mathrm{s}}}{\partial n} = -k_{\mathrm{f}} \frac{\partial T_{\mathrm{f}}}{\partial n} + h_r(T_{\mathrm{s}} - T_\infty) \tag{16}$$

where h_r is defined as follows:

$$h_r = \varepsilon\sigma\left(T_s^2 + T_\infty^2\right)(T_{\mathrm{s}} + T_\infty) \tag{17}$$

A line-by-line TDMA iterative method (Patankar, 1980) was used to solve the discretized governing (algebraic) equations. For each iteration sweep, h_r is calculated using the values of T_{s} obtained in the previous iteration level. To avoid eventual divergence of the iterative process, the under-relaxation factors of 0.5, 0.5, 0.8, and 0.8 for u, v, p and T, respectively, were selected based on numerical experiments.

The discretized governing equations are solved for three different computational domain sizes (0.15×0.03 m, 0.15×0.05 m, and 0.15×0.07 m) which the maximum Grashof number, based on the heat input rate, in all of tests is less than the critical Grashof number to keep regime of flow to be laminar.

The convergence criterion, based on the maximum relative residual of the discretized equations, is chosen as 0.01. The maximum relative residual of the discretized equations is defined as the ratio between the maximum residual for every variable in the computational domain in each iteration and the same value for the second iteration.

4. Discussion of results

As a first step in the assessment of the accuracy of the predictions of the present numerical model the results of the simplified steady-state governing equations with constant value for all physical and thermo-physical properties of the fluid (air) except density (Boussinesq approximation) were calculated, as shown in Figures 2-4.

Figure 2 shows the current computational results for iso-ξ coordinates based on the definitions of Fujii (a) and Gebhart *et al.* (b), in the computational domain for $Q_{tot} = 70$ w/m. Fujii (1963) defined the Grashof number based on the total heat rate from heat source of plume, where as Gebhart *et al.* (1970) defined the Grashof number based on the temperature difference between the centerline (symmetry line) of the plume and the ambient. Details of these definitions, including ξ, are presented in the Appendix. The assumption of boundary layer flow (thin layer) for high values of ξ seems to be more reasonable for the definition of Fujii (1963) than that presented by Gebhart *et al.* (1970).

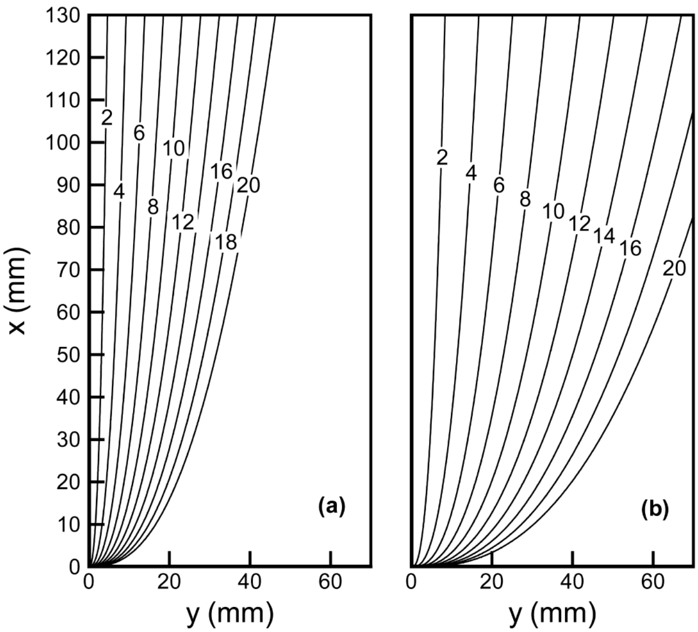


Figure 2.
Configuration of iso-ξ contours based on:
(a) Fujii (1963);
(b) Gebhart *et al.* (1970)

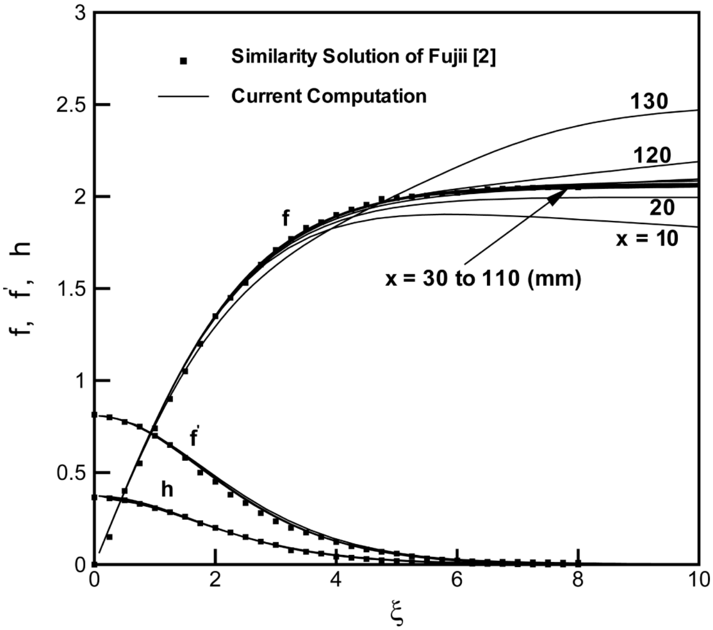


Figure 3.
Comparison of the current computation with the resuls of Fujii (1963) based on the boundary layer assumptions

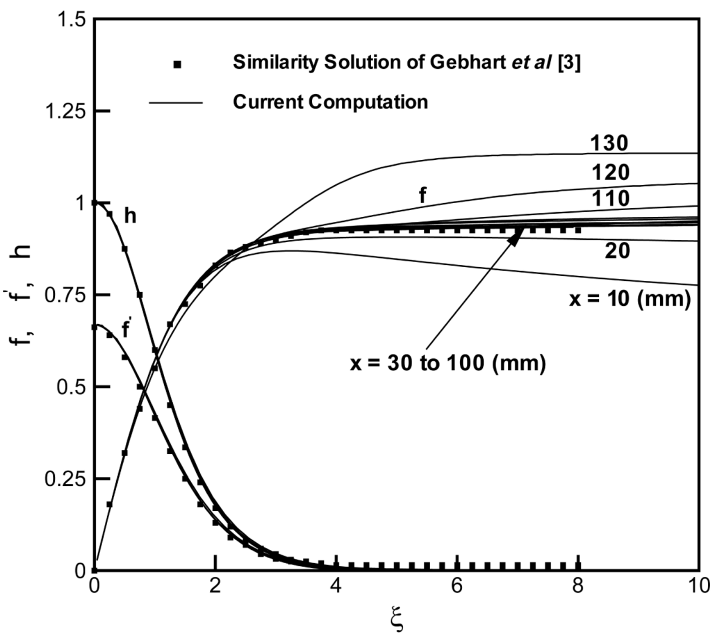


Figure 4.
Comparison of the current computation with the results of Gebhart *et al.* (1970) based on the boundary layer assumptions

Figure 3 shows the numerical results for the steady-state Boussinesq equations, and they compare well with the results of the similarity solution of Fujii (1963), for the boundary layer equations, particularly within the thin layer, where the assumptions made are valid. This is corroborated by comparing the results for the variables f, f', and h. They are plotted for different heights from the center of heat source (x varies from 10 to 130 mm by 10 mm steps). The boundary layer assumption does not consider the momentum equation in the y direction, hence the pressure gradient in this direction is assumed to be equal to zero. In this work, however, the momentum equation in the y direction was solved. This is the main difference between the boundary layer theory and the current computational model. There is good agreement between the similarity solution of Fujii (1963) and the present predictions except for f in the regions of $x < 30$ mm and $x > 110$ mm. The region of $x < 30$ mm is very close to the heat source (leading edge), and the boundary layer assumption is not satisfied. This may explain the discrepancy in the results for the high end values of ξ, since in this region y is larger than x (Figure 2). The solution of the momentum equation in the y direction, and the existence of a pressure gradient in this direction may yield the observed discrepancies. These reasons are also valid for the results in the region of $x > 110$ mm, however, the two most common definitions of the Grashof number only affect the display of the results; however, Fujii's definition may be preferred, considering the slenderness of the boundary layer.

Figure 4 reports on the present predictions along with the results of Gebhart *et al.* (1970). The two sets are in good agreement with each other, however, the discrepancies shown in Figure 3, are also present in a similar range of x and ξ. Taking into account the two different definitions for Gr, ξ, f, f' and h (Appendix), the current numerical predictions for the Boussinesq equations lend support to the validity of the analytical-numerical (similarity) solutions. The comparison of the results (Figures 3 and 4) show that the solution of the y momentum, as expected, has no significant effect on the results for small value of ξ, where the boundary layer assumption is valid. For large values of $\xi(\xi > 3)$, however, the assumption of zero gradient pressure is not applicable, therefore, the y momentum equation cannot be neglected, as clearly shown in Figures 3 and 4.

In addition, Figure 3 shows that the f curves in the region of 30 mm $< x <$ 110 mm for higher value of ξ are in better agreement with the present predictions than those shown in Figure 4. Tentatively, this may indicate the formulation of Fujii (1963) has broader applicability than that of Gebhart *et al.* (1970), particularly for higher values of ξ.

The mesh convergence of the results was also carefully analysed. The effect of width of computational domain and mesh size on the results is shown in Figure 5. For this purpose, the dimensions of the computational domain were selected as $30 \times 150\,\text{mm}^2$, $50 \times 150\,\text{mm}^2$,and $70 \times 150\,\text{mm}^2$, with meshes of 81×193, 131×193, and 181×193, respectively. The height of the computational domain (H) was chosen to be 150 mm, with the purpose of restricting the Grashof number at that location, to a value below 10^8, which is the approximate value for the transition from the laminar to the turbulent regime. The non-dimensional variables f, f' and h in the region of 30 mm $< x <$ 110 mm are independent of x (Figures 2 and 3), therefore, only the results for $x = 90$ mm are shown in Figure 5. The width of the computational domain (W) affects the results for $W < 50$ mm, the data clearly show that for $W < 50$

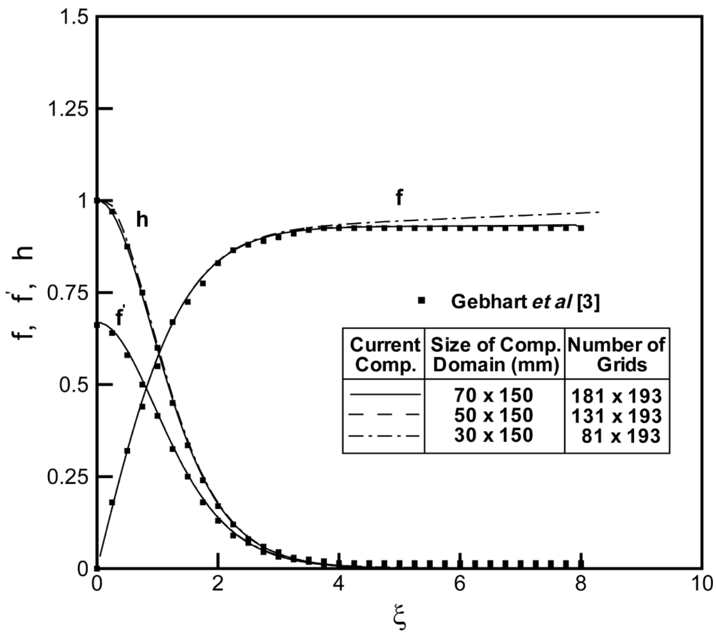


Figure 5.
The effects of computational domain and mesh size on the results for a fixed value of $x (x = 90\,\mathrm{mm})$

the pressure constant boundary condition imposes undue flow constraints, which are apparent by the "narrowing" of the plume due to its restricted transversal diffusion. For higher values of the width ($W > 50\,\mathrm{mm}$), the results are independent of the size of W. This is important as the use of a non-uniform mesh yields different number of meshes for each domain size. In what follows the effect of assumptions required by the boundary layer theory (Appendix) are examined.

One of the assumptions made in the boundary layer theory is to take as zero the second derivative of variables (u and T) in the vertical direction (x) (Gebhart *et al.*, 1988). The Boussinesq equations with zero second derivatives in the x direction were solved numerically and the results are shown in Figure 6. Comparison of Figures 4 and 6 shows negligible effect of these terms, especially for low values of ξ, but in the region of high value of $\xi (\xi > 3)$, their effect on the results can be noticed. This observation makes good physical sense in the region of high value of ξ (Figure 2), the order of magnitude of the second derivative with respect to x has the same order of magnitude of that with respect to y. Therefore, to neglect it is not warranted.

Another factor that may affect the results is the variation of physical and thermo-physical properties of the fluid with the temperature. The variation of temperature in this study is high (about 100 K), and an analysis based on work of Gray and Giorgini (1976) indicates that it is not valid to make the Boussinesq approximation. The effect of the variation of density with temperature (equation (9)) on the results at two different heights ($x = 30$ and $90\,\mathrm{mm}$) based on the definitions of Fujii (1963) and of Gebhart *et al.* (1970) are shown in Figures 7 and 8, respectively. For these cases, only the variation of density with temperature was considered for the body force term, and advective terms. The other properties μ, c_p, Pr and k, are assumed constant. Figures 7 and 8, as expected, show that the variation of density with temperature has a more

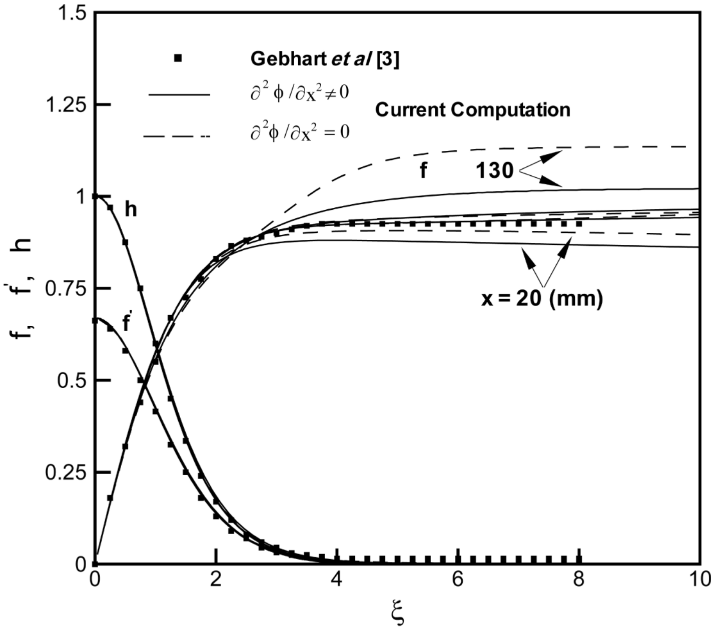
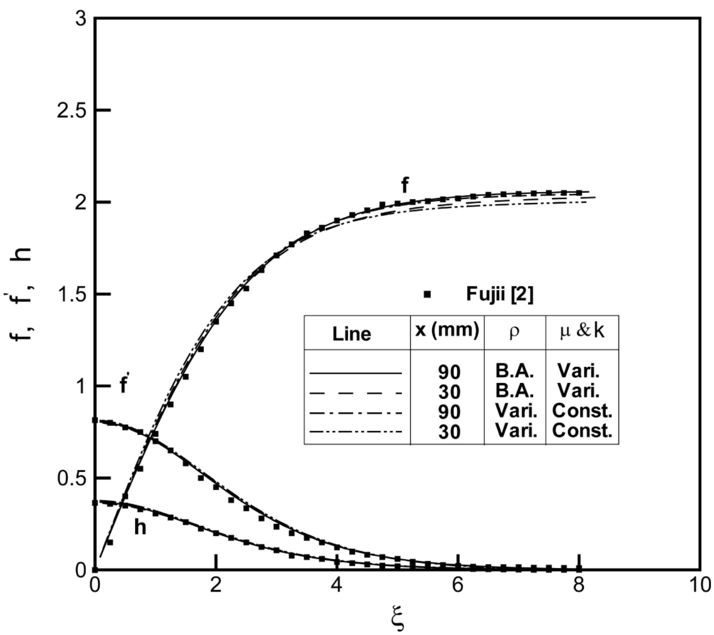
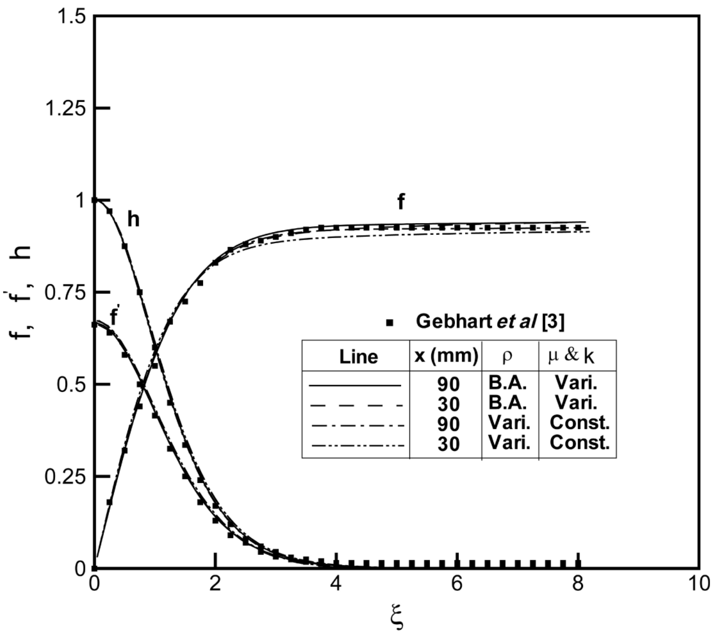


Figure 6.
The effect of second derivative of variables in the governing equations on the results



Notes: B.A.: Boussinesq approximation; Vari: Dependent on the temperature; Const: Constant

Figure 7.
The effect of temperature-dependent properties of the fluid upon the predictions, and their comparison against the results of Fujii (1963)



Notes: B.A.: Boussinesq approximation; Vari: Dependent on the temperature; Const: Constant

Figure 8. The effect of temperature-dependent properties of the fluid upon the predictions, and their comparison against the results of Gebhart *et al.* (1970)

pronounced influence upon the results in the vicinity of the heat source ($x = 30$ mm) than in the far field region ($x = 90$ mm). In the region close to the heat source, the temperature of the fluid is much higher than the far field temperature, and well beyond the range of applicability of the Boussinesq approximation. In another test case, it is assumed the Boussinesq approximation is only partially satisfied, c_p and Pr are constant, but μ and k vary with the temperature in accordance with equations (10) and (11), respectively. The influences of these parameters upon the results are also shown in Figures 7 and 8. The variation of the viscosity and thermal conductivity of the fluid (air) with temperature has a far less pronounced effect than that caused by the variation of density.

One of the possible reasons for the discrepancy between the analytical solutions and the experimental results as mentioned in Gebhart *et al.* (1988), is the net radiation transfer between the surface of the heat source ($\varepsilon = 1$) and the surroundings. Figure 9 shows the effect of this factor on the results. It shows that if the variables f, f', and h are to be normalized with respect to the total power of the heat source, the values of f and f' are then very different from those obtained with the similarity solution method. However, if the variables are normalized with respect to the amount of heat, which is transferred by convection (total heat minus radiation lost from the surface of the heat source to the surroundings), the values of f and f' then coincide with the similarity solution results. This means that the similarity solution only takes into consideration the convection heat transfer.

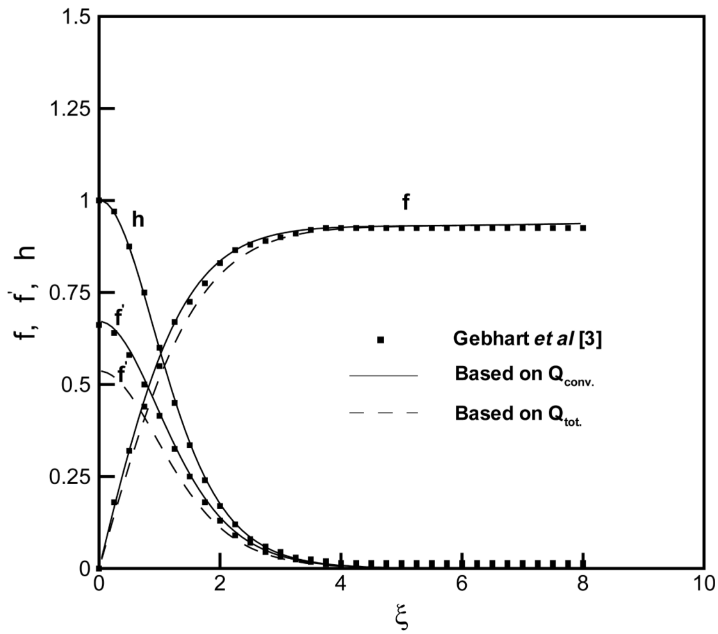


Figure 9.
The effect of radiation from the heat source (solid) upon the results

The overall combined effect of all assumptions embodied by the boundary layer equations (Appendix) upon the results for the laminar steady-state thermal plume is analysed by solving the steady-state "full" Navier-Stokes equations. The changes of density, viscosity, and thermal conductivity were calculated from equations (9)-(11), respectively. The present results for this case are shown in Figure 10. It shows that there is no agreement between the present predictions and the similarity solutions, even for the values of f' and h. The effect upon the predictions by each individual assumption is small as compared to that when the assumptions are combined.

The distribution of the centerline temperature in the x direction is an important variable characterizing the plume, as extensively discussed in the literature. Gebhart *et al.* (1988) investigated the variation of the non-dimensional steady-state laminar centerline temperature of the plume with respect to the Grashof number for different experimental data. They compare the experimental data against the results of the similarity solution of boundary layer equations. The non-dimensional centerline temperature of plume is defined as:

$$T^* = (T(x,0) - T_\infty)4\sqrt{2}\frac{\mu c_p \mathrm{I}}{Q} \tag{18}$$

where

$$\mathrm{I} = \int_{-\infty}^{\infty} f'h \,\mathrm{d}\xi \tag{19}$$

The integral of equation (19) depends only on the Prandtl number and it is equal to 1.245 for $Pr = 0.7$ (Gebhart *et al.*, 1988). The experimental results described by T^* are

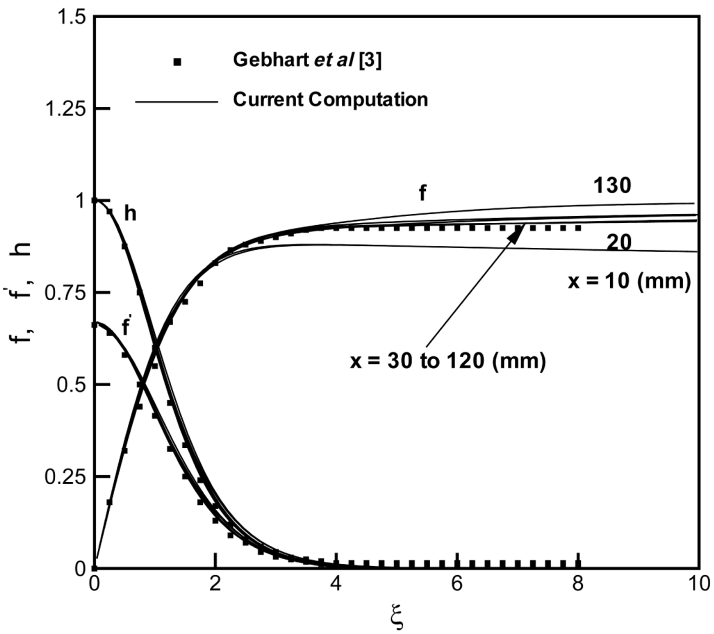


Figure 10.
Numerical results without resorting to the Boussinesq assumptions

about 15 percent lower than the boundary layer theory (Gebhart, 1973). The present predictions for this quantity with radiation ($\varepsilon = 1$) and without radiation ($\varepsilon = 0$) are compared in Figure 11 against the experimental results, and against the results derived from the boundary layer theory. For all test cases, when the radiation from the heat source is not considered, the agreement between the present predictions and the similarity solutions is good. When radiation between the heat source and the surroundings is taken into account, the predictions then move closer to the experimental results. It should be mentioned that in this case, if the amount of heat by convection is inserted in equation (18) instead of the total heat, the result will agree well with the similarity solutions. Comparison between the experimental results (Brodowicz and Kierkus, 1966; Forstrom and Sparrow, 1967; Schorr and Gebhart, 1970), and the current predictions allows to conclude that one of the parameters that causes major discrepancy between the experimental centerline temperature of plume and the predicted values by the similarity solutions is the net radiation exchange between the heat source and the surroundings. In Forstrom and Sparrow (1967), the heater was covered with a thin layer of polished gold, however, it seems that even a small amount of radiative exchange between the heat source and the surroundings may have an important effect upon the centerline temperature of the plume.

5. Conclusion

In this study, the analysis of a two-dimensional laminar thermal plume induced by a horizontal line heat source is conducted by numerical techniques associated with the simple method. The computational results are presented for a range of Grashof numbers and are for a variety of simplifying assumptions of the governing equations.

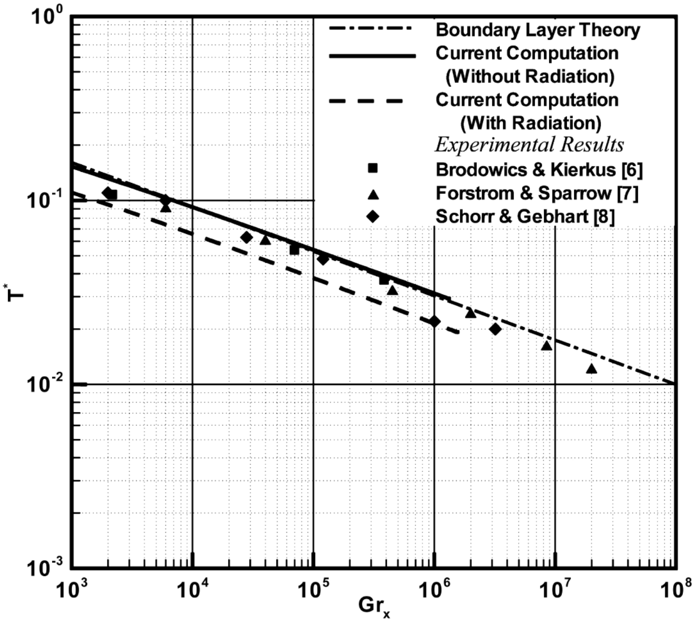


Figure 11.
The variation of non-dimensional centerline temperature of plume with respect to Grashof number

The Boussinesq equations show good agreement with two different similarity solutions.

When the Boussinesq approximation is not made, it was verified that the variation of the fluid density with the temperature in the advective terms in addition to the body force term has a much larger effect upon the predictions than that induced by the variation of thermal conductivity or viscosity with temperature.

The analysis indicates that a possible explanation for the discrepancy between the experimental results and the similarity solutions is the exchange radiation between the heat source and the surroundings. Moreover, the non-dimensional centerline temperatures of the plume obtained through the similarity solutions, and normalized using the convection heat transfer instead of total source heat output, are in good agreement with the experimental values.

References

Anderson, J.D. Jr (1991), *Fundamentals of Aerodynamics*, 2nd ed., McGraw-Hill, New York, NY.

Brodowicz, K. and Kierkus, W.T. (1966), "Experimental investigation of laminar free convection flow in air above horizontal wire with constant heat flux", *Int. J. Heat Mass Transfer*, Vol. 9, pp. 81-94.

Esfahani, J.E. and Sousa, A.C.M. (1999), "Ignition of epoxy by a high radiation source: a numerical study", *Int. J. Thermal Science*, Vol. 38, pp. 315-23.

Forstrom, R.J. and Sparrow, E.M. (1967), "Experimental on the buoyant plume above a heated horizontal wire", *Int. J. Heat Mass Transfer*, Vol. 10, pp. 321-31.

Fujii, T. (1963), "Theory of the steady laminar natural convection above a horizontal line heat source and a point heat source", *Int. J. Heat Mass Transfer*, Vol. 6, pp. 597-606.

Gebhart, B. (1973), "Natural convection flows and stability", *Advances in Heat Transfer*, Vol. 9, pp. 273-346.

Gebhart, B., Pera, L. and Schorr, A.W. (1970), "Steady laminar convection plumes above a horizontal line heat source", *Int. J. Heat Mass Transfer*, Vol. 13, pp. 161-71.

Gebhart, B., Jaluria, Y., Mahajan, R.L. and Sammakia, B. (1988), *Buoyancy Induced Flows and Transport*, Hemisphere, New York, NY.

Gray, D.D. and Giorgini, A. (1976), "The validity of the Boussinesq approximation for liquids and gases", *Int. J. Heat Mass Transfer*, Vol. 19, pp. 545-51.

Incropera, F.P. and DeWitt, D.P. (2002), *Fundamentals of Heat and Mass Transfer*, 5th ed., Wiley, New York, NY.

Jaluria, Y. and Gebhart, B. (1977), "Buoyancy induced flow arising from a line thermal source on an adiabatic vertical surface", *Int. J. Heat Mass Transfer*, Vol. 20, pp. 153-7.

Kuehn, T.H. and Goldstein, R.J. (1980), "Numerical solution to the Navier-Stokes equations for laminar natural convection about a horizontal isothermal circular cylinder", *Int. J. Heat Mass Transfer*, Vol. 23, pp. 971-9.

Lin, H-T., Chen, J-J., Kung, L-W., Yu, W-S. and Chen, Y-M. (1996), "Inclined and horizontal wall plumes", *Int. J. Heat Mass Transfer*, Vol. 39, pp. 2243-52.

Linan, B.A. and Kurdymov, V.N. (1998), "Laminar free convection induced by a line heat source, and heat transfer from wires at small Grashof numbers", *J. Fluid Mech.*, Vol. 362, pp. 199-227.

Patankar, S.V. (1980), *Numerical Heat Transfer and Fluid Flow*, McGraw-Hill, New York, NY.

Schorr, A.W. and Gebhart, B. (1970), "An experimental investigation of natural convection wakes above a line heat source", *Int. J. Heat Mass Transfer*, Vol. 13, pp. 557-71.

Shin, S-C. and Chang, K-S. (1989), "Transient natural convection heat transfer from a horizontal circular cylinder", *Int. Comm. Heat Mass Transfer*, Vol. 16, pp. 803-10.

Versteeg, H.K. and Malalasekera, W. (1995), *An Introduction to computational Fluid Dynamics, the Finite Volume Methods*, Prentice-Hall, Malaysia.

Wang, P., Kahawita, R. and Nguyen, D.L. (1991), "Transient laminar natural convection from horizontal cylinders", *Int. J. Heat Mass Transfer*, Vol. 34, pp. 1429-42.

Zeldovich, Y.B. (1937), "Limiting laws of freely rising convection currents", *Zh. Eksp. Teor. Fiz*, Vol. 7, pp. 1463-5.

Appendix

The main assumptions required by boundary layer theory for a laminar thermal plume are as follows:

- the governing equations were simplified for a thin region along the centerline of plume by assuming the transversal pressure gradient is negligible; and
- all physical properties of the fluid are taken constant, except for the density in the buoyancy force term in the momentum equation (Boussinesq approximation).

Using these assumptions, the simplified governing equations are:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(A1)}$$

Momentum equation in the x direction

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho g\beta\Delta T+\mu\frac{\partial^2 u}{\partial y^2} \tag{A2}$$

Energy equation

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2} \tag{A3}$$

Fujii (1963) defined the Grashof number as follows:

$$Gr=\frac{g\beta x^3\theta_0}{\nu^2} \tag{A4}$$

where

$$\theta_0=\frac{Q_{\mathrm{tot}}}{c_P\mu} \tag{A5}$$

which is function of x. Also, the following variables are defined as:

$$\xi=Gr^{1/5}\frac{y}{x},\quad \psi=\nu Gr^{1/5}f(\xi),\quad T-T_\infty=Gr^{-1/5}\theta_0 h(\xi) \tag{A6}$$

Equations (A1)-(A6) are combined and yield the following ODEs:

$$f'''+\frac{3}{5}ff''-\frac{1}{5}f'^2+h=0 \tag{A7}$$

$$h''+\frac{3}{5}Pr(fh)'=0 \tag{A8}$$

where

$$\frac{ux}{\nu}=Gr^{2/5}f',\quad \frac{vy}{\nu}=-\xi\left(\frac{3}{5}f-\frac{2}{5}\xi f'\right) \tag{A9}$$

The ODEs were numerically solved using appropriate boundary conditions.

Gebhart *et al.* (1970) defined the Grashof number based on the difference between the centerline temperature of the plume (T_0) and the ambient temperature ($T\infty$) as follows:

$$Gr=\frac{g\beta x^3(T_0-T_\infty)}{\nu^2} \tag{A10}$$

The Grashof number is a function of the x direction and the centerline temperature difference of the plume, which is also a function of x. The similarity parameters, stream function and non-dimensional temperature, were defined as:

$$\xi=\frac{y}{x}\sqrt[4]{\left(\frac{Gr}{4}\right)},\quad \psi=4\nu\sqrt[4]{\left(\frac{Gr}{4}\right)}f(\xi),\quad h(\xi)=\frac{T-T_\infty}{T_0-T\infty} \tag{A11}$$

Equations (A1)-(A3) combined with equations (A10) and (A11) yield the following ODEs:

$$f'''+\frac{12}{5}ff''-\frac{4}{5}f'^2+h=0 \tag{A12}$$

$$h'' + \frac{12}{5}Pr(fh)' = 0 \tag{A13}$$

where

$$u = 4^{1/5}\left(\frac{g\beta Q}{c_p \mathrm{I}}\right)^{2/5}\left(\frac{x}{\mu\rho}\right)^{1/5} f', \quad v = 4^{3/4}\frac{\nu}{x}Gr^{1/4}\left(\frac{3}{5}f - \frac{2}{5}\xi f'\right) \tag{A14}$$

In these equations the integral I can be calculated from:

$$\mathrm{I} = \int_{-\infty}^{\infty} f' h \, \mathrm{d}\xi \tag{A15}$$

which, as discussed for equation (17), is a function of the Prandtl number only, and for air ($Pr = 0.7$) is equal to 1.245. These ODEs are also solved numerically using the appropriate boundary conditions.

Corresponding author
Javad A. Esfahani can be contacted at abolfazl@um.ac.ir